Precalc Warm Up # 2-3

1. Write the equation of line $\perp$ to $4x - 3y = 9$ that goes through (0,8)

2. Solve the system for x and y:

$$ax + by = a - b$$
$$bx + ay = a - b$$

HW Questions? p. 37 & 38

3. Find the values of m such that these equations have no solutions.

(iii) $-3(4x - 2y = 12)$
$4(3x + my = 2)$

$m = -\frac{3}{2}$

Plan: Rewrite in slope-int form & compare.

$\begin{cases} \text{Same slope} \\ \text{Different y-ints} \end{cases}$

OR use combination:

$0x + 0y \neq \#$ not zero

4. Find the values of m and a such that these equations have infinite solution sets.

(iii) $3x + my = a$
$2x - 4y = 6$

Same line so make them match!

iii) $2(3x + my = a)$
$3(2x - 4y = 6)$

$6x + 2my = 2a$
$6x - 12y = 18$

x terms match, now look at the y-terms;

$2my = -12y$

$2m = -12$

$\boxed{m = -6}$

5. Find the solution sets of the following simultaneous equations, solving for x and y.

(b) $bx + y = a$
$ax + y = b$

(c) $ax + by = 1$
$ax - by = 1$

5. Find the solution sets of the following simultaneous equations, solving for x and y.

(e) $a(ax + by = a - b)$
$-b(bx + ay = a - b)$

$a^2x + aby = a^2 - ab$
$-b^2x - aby = b^2 - ab$

$a^2x - b^2x = a^2 - 2ab + b^2$

$\frac{x(a^2 - b^2)}{a^2 - b^2} = \frac{(a-b)^{\cancel{2}}}{(a+b)\cancel{(a-b)}}$

$x = \frac{a-b}{a+b}$

Now plug in to find y.

(f) $(ax + y = b)(-a)$
$bx + ay = 2ab - a^3$

$-a^2x - ay = -ab$

$bx - a^2x = ab - a^3$

$\frac{x\cancel{(b - a^2)}}{\cancel{b - a^2}} = \frac{a\cancel{(b - a^2)}}{\cancel{b - a^2}}$

$x = a$

$a(a) + y = b$

$y = b - a^2$

$\boxed{(a, b - a^2)}$ $b \neq a^2$

1. Solve the simultaneous equations by hand: best choice is to eliminate z.

(a) ① $4(6x + 4y - z = 3) \rightarrow 24x + 16y - 4z = 12$
② $x + 2y + 4z = -2 \rightarrow x + 2y + 4z = -2$
③ $5x + 4y = 0$

$25x + 18y = 10$

③ $\rightarrow (5x + 4y = 0)(-5)$

$4①+②$ $25x + 18y = 10$
$-25x - 20y = 0$

$-2y = 10$

$y = -5$

$\boxed{(4, -5, 1)}$

1.

(c)
$$\begin{aligned} 4x + 9y + 13z &= 3 \\ -x + 3y + 24z &= 17 \\ 2x + 6y + 14z &= 6 \end{aligned}$$

(d)
$$\begin{aligned} x - 2y - 3z &= 3 \\ x + y - 2z &= 7 \\ 2x - 3y - 2z &= 0 \end{aligned}$$

(e)
$$\begin{aligned} x - y - z &= 2 \\ 3x + 3y - 7z &= 7 \\ x + 2y - 3z &= 3 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & -1 & 2 \\ 3 & 3 & -7 & 7 \\ 1 & 2 & -3 & 3 \end{bmatrix} \rightarrow \text{rref} \rightarrow \begin{bmatrix} 1 & 0 & -5/3 & 0 \\ 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x - \frac{5}{3}z = 0$$

$$y - \frac{2}{3}z = 0$$

$$0 \neq 1$$

No Solution

Groups:

$$\begin{aligned} x - 2y &= -1 \\ -x - y + 3z &= 1 \\ y - z &= 0 \end{aligned}$$

rref:

$$\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Groups:

$$
\begin{aligned}
x - 2y &= -1 \\
-x - y + 3z &= 1 \\
y - z &= 0
\end{aligned}
$$

rref:

$$
\begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

Solution Generator

$x - 2z = -1$

$x = 2z - 1$

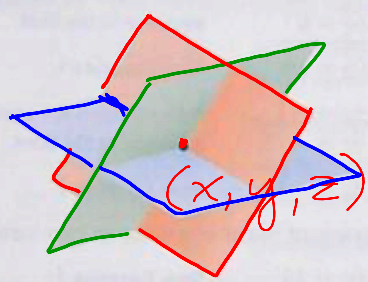
$y - z = 0$

$y = z$

$0 = 0 \leftarrow \infty$ solutions

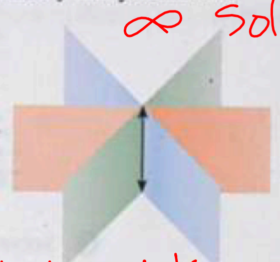
$(2z-1, z, z)$

If the planes intersect in a single point, as shown below, the system has exactly one solution.



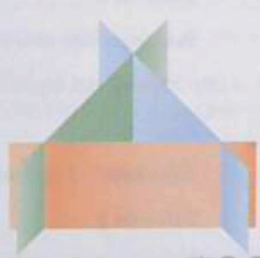
If the planes intersect in a line, as shown below, the system has infinitely many solutions.

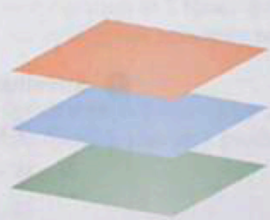
∞ Sol.



All the points on the line

If the planes have no point of intersection, the system has no solution. In the example on the left, the planes intersect pairwise, but all three have no points in common. In the example on the right, the planes are parallel.





$$\left(\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 2 \end{array}\right)$$ means

$x - 5z = 0$

$y - 2z = 3$

No Solution $0 \neq 2$

$$\left(\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$ means

$x - 5z = 0$

$y - 2z = 3$

$0 = 0$

∞ solutions

$(5z, 2z+3, z)$

solution generator

Review of Quadratics

Three methods used to solve Quadratic Equations: $ax^2 + bx + c = 0$

1. Completing the Square
2. Quadratic Formula
3. Factoring and using the Null Factor Law

We have called this the Zero Product Property

Ex: $(x+2)(x-3) = 0$

$x+2=0 \quad x-3=0$

$x = -2, 3$

Solving a quadratic by COMPLETING THE SQUARE

getting from here $\rightarrow$ $4x^2 - 11x - 3 \qquad = 0$

to here $\rightarrow$ $(x - \frac{11}{8})^2 \qquad = \frac{169}{64}$

Then $\sqrt{\ }$ both sides:

$$x - \frac{11}{8} = \pm \frac{13}{8}$$

$$x = \frac{11}{8} \pm \frac{13}{8}$$

$$x = 3, -\frac{1}{4}$$

Solve a quadratic by COMPLETING THE SQUARE

$ax^2 + bx + c = 0$

$4x^2 - 11x - 3$	$= 0$	set equation equal to 0
$4x^2 - 11x$	$= 3$	move constant to the other side
$4(x^2 - \frac{11}{4}x \qquad)$	$= 3$	factor out leading coefficient
$4(x^2 - \frac{11}{4}x + \frac{121}{64})$	$= 3 + 4(\frac{121}{64})$	complete the square and BALANCE the equation
$\frac{1}{4} \cdot 4(x - \frac{11}{8})^2$	$= \frac{169}{16} \cdot \frac{1}{4}$	simplify
		start isolating x
$(x - \frac{11}{8})^2$	$= \frac{169}{64}$	
$x - \frac{11}{8}$	$= \pm \frac{13}{8}$	
x	$= \frac{11}{8} \pm \frac{13}{8}$	
x	$= \frac{24}{8}$ or $\frac{-2}{8}$	
x	$= 3$ or $\frac{-1}{4}$	

Solve this quadratic equation by completing the square

$3x^2 + 24x - 5 = 0$

get constant on the other side

factor out leading coefficient

complete the square and
BALANCE the equation.

simplify

start isolating x

Solve this quadratic equation by completing the square

$3x^2 + 24x - 5 = 0$

$3x^2 + 24x = 5$

$3(x^2 + 8x + 16) = 5 + 48$

$(x^2 + 8x + 16) = \frac{53}{3}$

$\sqrt{(x+4)^2} = \pm\sqrt{\frac{53}{3}}$

$x + 4 = \pm\sqrt{\frac{53}{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$x = -4 \pm \frac{\sqrt{159}}{3}$

get constant on the other side

factor out leading coefficient

complete the square and
BALANCE the equation.

simplify

start isolating x

Solve by factoring:

$4x^2 - 11x = 3$

$4x^2 - 11x - 3 = 0$

$(4x+1)(x-3) = 0$

$\boxed{x = -\frac{1}{4}, 3}$

Solve: $x\left(x + \frac{20}{x}\right) = 9x$

$x^2 + 20 = 9x$

$x^2 - 9x + 20 = 0$

$(x-4)(x-5) = 0$

$\boxed{x = 4, 5}$

Solving quadratic equations using the QUADRATIC FORMULA

$ax^2 + bx + c = 0$

Solving this by completing the square gives us the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

A quadratic equation can have 1, 2 or no solutions.

Example with one solution: $4x^2 - 12x + 9=0$

$$x = \frac{12 \pm \sqrt{144 - 4 \cdot 4 \cdot 9}}{8}$$

$$x = \frac{12 \pm \sqrt{0}}{8}$$

$$\boxed{x = \frac{3}{2}}$$

Example with two solutions: $4x^2 - 15x + 9=0$

$$x = \frac{15 \pm \sqrt{225 - 4 \cdot 4 \cdot 9}}{8}$$

$$x = \frac{15 \pm \sqrt{81}}{8}$$

$$x = \frac{15 \pm 9}{8}$$

$$x = \frac{24}{8} \text{ or } \frac{6}{8}$$

$$\boxed{x = 3 \text{ or } \frac{3}{4}}$$

Example with no solutions: $4x^2 - 5x + 9 = 0$

$$x = \frac{5 \pm \sqrt{25 - 4 \cdot 4 \cdot 9}}{8}$$

$$x = \frac{5 \pm \sqrt{-119}}{8}$$

$$\boxed{\varnothing}$$

How can you tell how many solutions the quadratic equation has without actually solving the problem?

discriminant: $b^2 - 4ac$

Check the discriminant!

Check the discriminant!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac$ is called the discriminant because it discriminates how many solutions there will be.

If $b^2 - 4ac < 0$, then there will be NO SOLUTIONS

if $b^2 - 4ac > 0$, then there will be TWO SOLUTIONS

if $b^2 - 4ac = 0$, then there will be ONE SOLUTION

Find the value of m for which the equation

$2x^2 + mx + 1 = 0$ has one real solution

$ax^2 + bx + c = 0$

$b^2 - 4ac = 0$

$m^2 - 4(2)(1) = 0$

$m^2 = 8$

$m = \pm 2\sqrt{2}$

Find the value of k for which the equation

$x^2 + 4x + k = 0$ has two real solutions.

$b^2 - 4ac > 0$

$4^2 - 4(1)(k) > 0$

$\frac{16}{4} > \frac{4k}{4}$

$\boxed{k < 4}$

HW: SL Book
p. 43 #1 - 4 LC, 5 - 9

Show the specified method!

Quiz tomorrow: SL 2.1 - 2.3